*Dissertation Summary*

# Data-driven analysis and control of nonlinear systems with stability and performance guarantees

## *A linear parameter-varying approach*

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Ever-increasing performance requirements – such as higher accuracy, energy efficiency, or increased speed – in fields like aerospace engineering, high-precision mechatronics, and energy distribution networks bring new challenges to the design of control systems. One of the main challenges is addressing the nonlinear regions of the system behavior, which is essential to meet such performance requirements. Particularly, for the successful design of high-performance controllers for the aforementioned systems, we need an accurate *representation* of their complex behavior. In practice, such a representation comprises a mathematical model that accurately describes the nonlinear system dynamics. This mathematical model is often obtained from first-principles knowledge. This means that expert engineers carefully analyze the complex system to accurately describe all the performance-critical aspects of the behavior, e.g., friction, aerodynamic effects, or hysteresis. Obtaining such an accurate model is therefore often a time-consuming, tedious, costly, and in some cases even an impossible task. For this reason, the mathematical model is often identified from measurement data (upper path in Figure 1). Identifying an accurate model from measurement data using state-of-the-art identification methods often results in non-negligible estimation errors, however. In fact, the development of accurate and systematic identification methods for complex nonlinear behaviors is still an active research area. Neglecting the uncertainties that result from the modeling or identification process can compromise stability and performance guarantees of the eventual controller design when it is deployed on the real system. On the other hand, incorporating (possibly conservative estimates of) the uncertainty in the design will lead to an inevitable trade-off between performance and robustness. To overcome this trade-off, recent research has focused on designing controllers *directly* from measured data (lower path in Figure 1), alleviating the problems in the modeling procedure and focusing directly on the control objective.

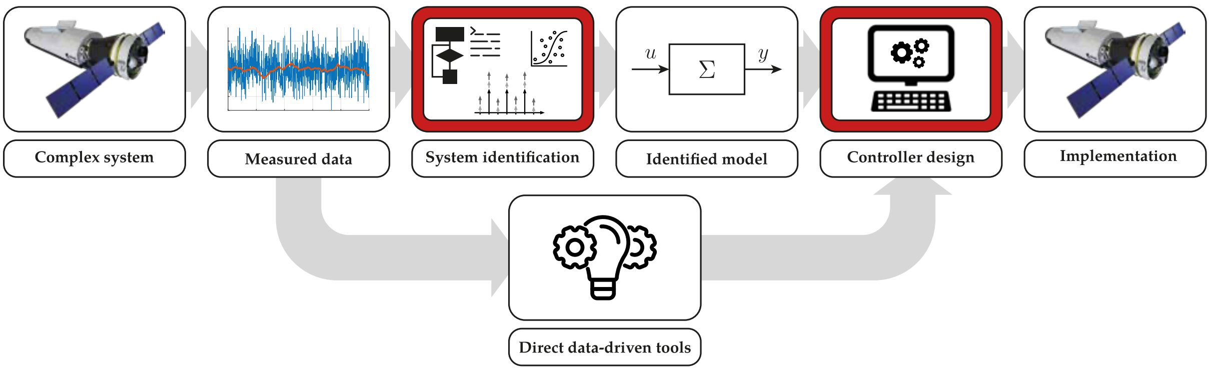


Figure 1: Two approaches for data-driven design: the indirect and the direct approach.

This thesis aims to provide a theoretical framework for direct data-driven analysis and control design of complex nonlinear systems with rigorous guarantees on stability and performance. Existing literature on direct data-driven control mainly focuses on methods that are applicable to systems with *Linear Time-Invariant* (LTI) behavior, while nonlinear data-driven methodologies available in the literature generally consider specific classes or approximations of nonlinear systems. In general these methods only provide local guarantees (i.e., guarantees that are valid in a neighborhood of an equilibrium point), and often do not provide guarantees on performance. Hence, it is still largely an open question how to achieve data-driven controller design for nonlinear systems with *equilibrium-free* stability and performance guarantees in practice.

In this thesis, we fill this gap in the literature. Our approach is based on two important ideas: First, we consider analysis and controller synthesis based on the so-called *velocity form*, describing the velocity dynamics of the nonlinear system. This form allows us to perform analysis and controller design with equilibrium-free performance and stability guarantees. The velocity form of a (non-)linear system is a representation that is inherently quasi-linear in the signals of the system. The second important idea is that we recognize that the velocity form can be embedded as a *Linear Parameter-Varying* (LPV) representation. LPV models represent dynamical systems through a linear relationship between the signals of the system, such as the input, output, and/or state. This relationship, however, is varying along a *measurable* signal, inducing the parameter-variations in the model. This signal is called the scheduling variable and is assumed to vary in a bounded set (generally representing the operating range of the system). The well-established model-based LPV framework extends the systematic and computationally efficient approaches of the LTI framework and has successfully been serving as a bridge between LTI and nonlinear systems. We develop these two main ideas in the context of the *behavioral approach*. The behavioral approach is a system-theoretical framework that views dynamical systems as a collection of system trajectories. This fits well in a data-driven context where we measure system trajectories, and thus offers a unified framework for providing stability and performance guarantees directly from data. Therefore, developing a behavioral framework for data-driven analysis and control of LPV systems has the potential of serving as a framework for achieving systematic and computationally efficient data-driven analysis and control of nonlinear systems with equilibrium-free guarantees. In this thesis, we develop exactly such a framework. Specifically, we pave the way towards direct data-driven methods of nonlinear systems in terms of: (*i*) Generalization of model-based LPV analysis and control methods to the data-driven case, and (*ii*) Developing formal, equilibrium-free guarantees for nonlinear systems from data through LPV embeddings of the velocity form.

The main research question in this thesis is formulated as follows: "*How to develop a data-driven analysis and control framework for nonlinear systems, capable of providing equilibrium-free stability and performance guarantees?*" To arrive at an answer to this question, we divide it up in three sub-research questions, providing the three main contributions in this thesis:

1. *How to obtain a data-driven representation of an LPV system?*
2. *How to achieve direct data-driven analysis and control of LPV systems?*
3. *How to achieve equilibrium-free stability and performance guarantees for nonlinear systems in a direct data-driven setting?*

The first main contribution of this thesis answers the first sub-question through the development of data-driven representations of LPV systems. Based on the behavioral approach for LPV systems, we present a representation of the finite-horizon behavior of an LPV system that is purely constructed from data. We achieve this for scenarios where only input-state, or only input-output data are available. Furthermore, we consider LPV systems with a general meromorphic, dynamic scheduling dependence, and LPV systems that have a so-called shifted-affine functional dependence on the scheduling signal. Particularly for the latter, the data-driven representation is directly constructible from data. Moreover, it is straight-forward to compute whether the data can represent the full finite-horizon behavior. The representations corresponding to this first main contribution of the thesis are foundational building blocks for data-driven simulation, analysis, and (predictive) control schemes for LPV systems.

Our answer to the second sub-research question provides several contributions to the literature on control theory and applications. We can summarize these contributions as the second main contribution of this thesis: The development of direct data-driven simulation, analysis and control tools for LPV systems with guarantees. We utilize the representations of our first main contribution to generalize existing data-driven tools for LTI systems to LPV systems. We develop a direct data-driven method for the simulation of LPV systems and generalize this solution to a data-driven interpolation approach for finding LPV system trajectories that interpolate missing data. These tools enable the development of methods for data-driven analysis and (predictive) control problems. Specifically, we develop methods for the direct data-driven dissipativity analysis of an LPV system from input-output or input-state-output data. Next to this, we present a set of efficient computational methods that allow for solving the analysis problems using off-the-shelf tools. In terms of LPV controller design, we develop data-driven methods for the synthesis of state- and output-feedback controllers. The data-based synthesis problems can be efficiently solved as a semidefinite program, and provide guarantees of the closed-loop system in terms of stability and performance (quadratic performance, $\ell_2$-gain, or $H_2$-norm-based performance metrics). The synthesis methods can handle both noise-free and noisy data. In fact, we show by means of a real-world experiment that the developed techniques are readily applicable for nonlinear systems through the use of LPV embeddings. As a final component of this main contribution, we develop *Data-driven Predictive Control* (DPC) methods for LPV systems. We present schemes for both the input-output setting and the input-state setting. The DPC schemes guarantee asymptotic stability of the closed-loop system and recursive feasibility of the optimization program, despite being only constructed from measurement data. We also provide a handful of guidelines for the use of the methods in practice, e.g., usage for nonlinear systems, handling noisy data, and computationally efficient implementations. All the developed simulation, analysis and control methods are validated in either a realistic simulation environment or on an experimental setup, where the to-be-controlled plant is a nonlinear system.

Together, these first two main contributions form a framework for *direct data-driven analysis and control of LPV systems with stability and performance guarantees*. Through the use of an LPV embedding of a nonlinear system, these methods are directly applicable to nonlinear systems, and thus inherently provide a framework for direct data-driven analysis and control of nonlinear systems. When the developed data-driven LPV methods are applied to nonlinear systems, however, the provided guarantees are *local*. That is, the stability and performance guarantees are valid in a neighborhood around the origin/point of natural storage of the nonlinear system. The final main contribution of this thesis allows us to go beyond local guarantees, and provide *equilibrium-free* guarantees, which is often required for high-performance reference tracking and/or disturbance rejection scenarios.

For the third main contribution, we study the concept of velocity forms in the data-driven context. Local guarantees of these velocity forms provide equilibrium-free guarantees for the original nonlinear systems in terms of *Universal Shifted Asymptotic Stability* (USAS) and *Universal Shifted Dissipativity* (USD). Hence, the development of control methods and data-driven representations of velocity forms of nonlinear systems allow us to answer our final sub-research question. First, we derive velocity forms for nonlinear input-output and input-state representations. Through the use of a known basis function expansion, we propose an LPV embedding of the velocity form, on which we apply the data-driven LPV methods that have already been developed in this thesis. This application directly provides the global guarantees in the case of data-driven analysis. In the case of data-driven controller design, a scheme is required that realizes the controller for the original nonlinear system. We develop a realization scheme that realizes the velocity controller for the original nonlinear system and preserves the provided guarantees. Hence, our scheme ensures equilibrium-free stability and performance guarantees of the closed-loop in terms of USAS and USD. This procedure, however, is only successful under the assumption that the basis functions that define the LPV embedding of the velocity form are known. As a second part of this main contribution, we propose kernel-based methods to directly obtain a data-driven representation of the velocity form, without the need to know these basis functions. The proposed formulation is data-efficient, highly flexible, and can be directly used for analysis and control through the methods developed in this thesis. The proposed kernel-based data-driven representation is applicable for both the original and velocity form of the nonlinear system in question. This development provides an essential building block for a direct data-driven analysis and control framework for nonlinear systems with *equilibrium-free* guarantees, and finally allows us to answer the main research question.

From the presented contributions, we can draw the following main conclusion of this thesis: *the data-driven LPV framework is a powerful and effective way of achieving analysis and control of nonlinear systems in a data-driven setting, and the proposed extension to achieve equilibrium-free guarantees falls naturally within this framework through the use of the velocity form of nonlinear systems*. Throughout the thesis, the developed theoretical contributions are demonstrated and validated on a wide range of academic examples, realistic simulations, and experimental applications with nonlinear systems. These demonstrations show that the proposed methods are applicable in real-world scenarios, achieve comparable or improved performance compared to model-based methods, and show significant improvements over the existing data-driven LTI methods. The practical relevance of the work in this thesis is backed up by providing computationally tractable methods, openly accessible code repositories, and real-world experimental implementations. The final part of this thesis recommends topics for future research that we consider most important and pressing, which are mainly on systematic noise handling, automated (kernelized) LPV embedding of nonlinear systems, and input design for informative data generation.